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14. ABSTRACT

This proposal concerns development, analysis, implementation, and applications of efficient and accurate numerical methods for interface problems, especially with moving interface and free boundary problems, and problems defined on irregular and infinite domains. Specifically, we will develop numerical methods for various applications.

15. SUBJECT TERMS

Sharpe Interface Methods, Immersed Interface Method, moving interface and free boundary, Navier-Stokes equations

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Report Title

Final report: Subspace Iteration and Immersed Interface Methods: Theory, Algorithm, and Applications

ABSTRACT

This proposal concerns development, analysis, implementation, and applications of efficient and accurate numerical methods for interface problems, especially with moving interface and free boundary problems, and problems defined on irregular and infinite domains. Specifically, we will develop numerical methods for various applications.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Number of Papers published in peer-reviewed journals: 13.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 6.00

(c) Presentations

Number of Presentations: 22.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 6

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 2

(d) Manuscripts

Number of Manuscripts: 4.00

Patents Submitted

Patents Awarded

Graduate Students

NAME	PERCENT_SUPPORTED
Yan Gong	0.10
Qunlei Jiang	0.10
Hui Xie	0.20
FTE Equivalent:	**0.40**
Total Number:	**3**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Zhilin Li	0.40	No
Kazufumi Ito	0.40	No
FTE Equivalent:	**0.80**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics
This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields: 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields: 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale): 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering: 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields: 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Yang Gong
Qunlei Jiang
Hui Xie

Total Number: **3**

Names of other research staff

NAME	PERCENT_SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

1 Statement of the problem studied and Significance

This proposal concerns development, analysis, implementation, and applications of efficient and accurate numerical methods for interface problems, especially with moving interface and/or free boundary problems, and problems defined on irregular and infinite domains. Specifically, we will develop numerical methods for fully elastic systems with composites and material interfaces, fully transient simulations of two phase flows, e.g., droplets deformation and breakups in shear flows of immiscible fluids, and electro-magnetics, elastic wave propagation in heterogeneous media.

2 Summary of the most important results

- We have developed a fast iterative solver for scattering by elastic objects in layered media.

- We have nice convergence analysis of the primal-dual active set strategy for diagonally dominant systems.

- We have developed a preconditioned iterative methods on sparse subspaces.

- We have developed new immersed finite-element methods for elliptic and elastic interface problems with homogeneous and non-homogeneous jump conditions [2, 3]. The new methods do not require body-fitted meshes. Non-homogeneous jump conditions are removed by a known function that has the same jump conditions as that of the solution via a level set function. The new approaches provide a second order discrete delta function for elliptic and elastic interface problems. The results have been published in SIAM Numer. Anal. [3].

- In the past, we derived jump conditions for the Stokes and Navier-Stokes equations with continuous viscosity [7, 8]. We have derived the jump conditions for discontinuous viscosity recently using the idea of

the immersed interface method in [11]. We also developed the immersed interface methods for various problems including new approach to deal with pressure jump conditions [13], the Stokes flows with discontinuous viscosity [10, 18], biharmonic equations on irregular domains [1], incompressible Navier-Stokes equations on fixed irregular domains [10] with application to flow past fixed obstacles.

- In the application to problems in mathematical biology, our immersed-interface/level set method applied to the biological problem of forces creating branching morphogenesis shows that contractility of the mesenchyme is indeed sufficient to create a realistic cleft in a branching embryonic rudiment of a glandular organ. This is quite possibly the first simulation ever to realistically model the forces in branching morphogenesis, [15].

- We have some detailed analysis for a mixed elliptic and hyperbolic system for modeling tissue deformation [6].

  With partial support of the ARO grant, we have published about 15 journal papers [5, 13, 10, 3, 11, 18, 1, 15, 14, 6, 4, 20, 12, 16, 19, 17], and one book [9] in the last three years.

3   Other contributions

Please see the list of publications of and other reported information in the on-line form of the final report.

In a summary, we are able to accomplish almost all the projective in our original proposal and more. Furthermore, the ARO grant really helped us to get our research projects going and move to new research areas. We really appreciate ARO for the support.

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